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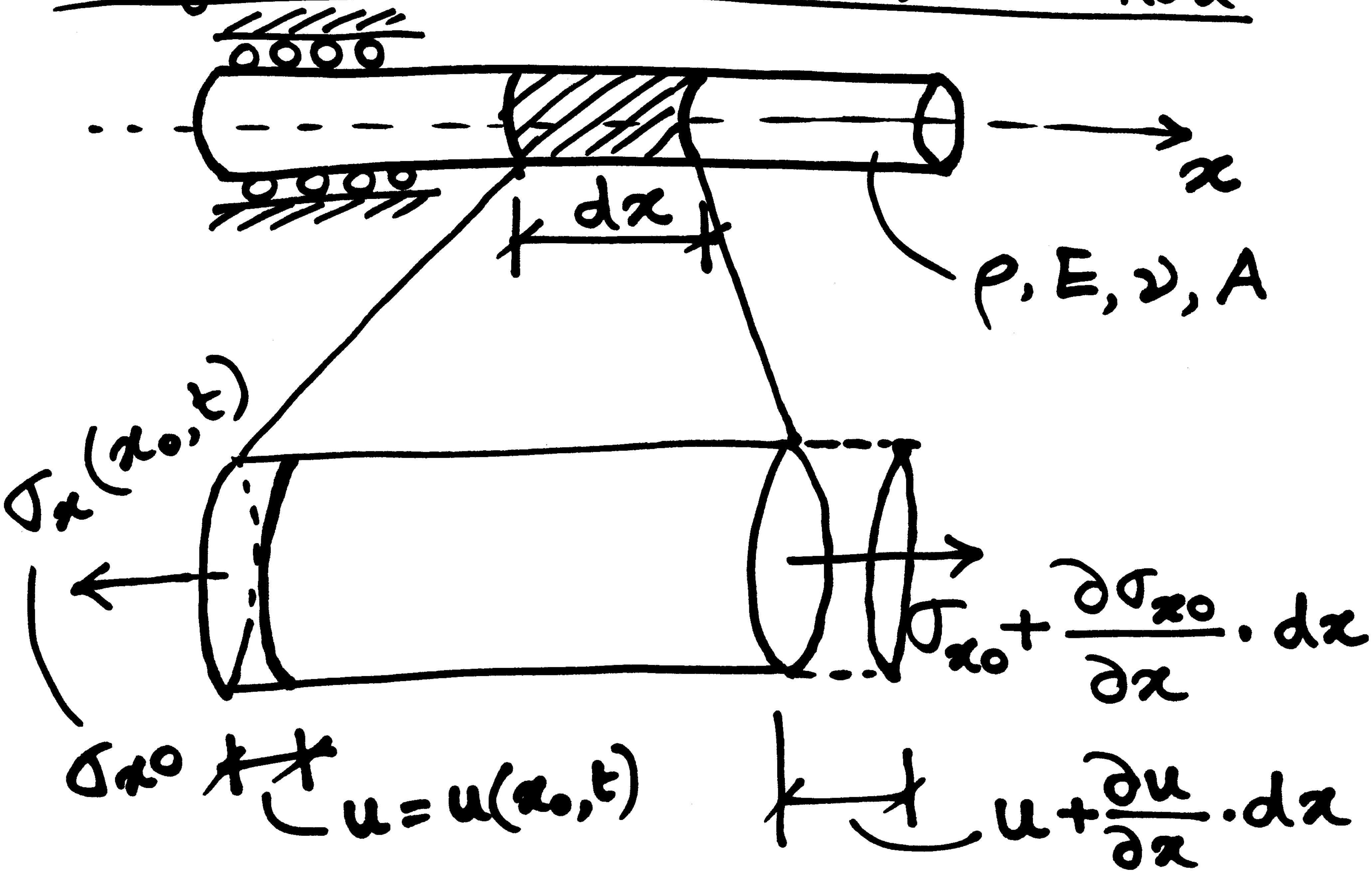
Longitudinal Wave in Infinite Rod



$$\left(\sigma_{x0} + \frac{\partial \sigma_x}{\partial x} \cdot dx\right) A - \sigma_{x0} \cdot A$$

$$= (\rho A \cdot dx) \cdot \frac{\partial^2 u}{\partial t^2}$$

or, $$\cancel{(\sigma_{x0} \cdot A)} + \frac{\partial \sigma_x}{\partial x} \cdot \cancel{dx} \cdot \cancel{A} - \cancel{(\sigma_{x0} \cdot A)}$$

$$= (\rho \cancel{A} \cancel{dx}) \cdot \frac{\partial^2 u}{\partial t^2} \quad \left(\because \begin{array}{l} dx \neq 0 \\ A \neq 0 \end{array}\right)$$

$$\boxed{\frac{\partial \sigma_x}{\partial x} = \rho \cdot \frac{\partial^2 u}{\partial t^2}}$$

Stress

Displacement

$$\sigma_x = M \epsilon_x \quad \text{(Stress-strain relationship)}$$

$$\epsilon_x = \frac{\partial u}{\partial x} \quad \text{(Strain-displacement relationship)}$$

$$M = \frac{(1-\nu)}{(1+\nu)(1-2\nu)} \cdot E$$

$\downarrow$

Constrained Modulus

$$\frac{\partial \sigma_x}{\partial x} = \rho \cdot \frac{\partial^2 u}{\partial t^2}$$

or, $$\frac{\partial^2 u}{\partial t^2} = \frac{1}{\rho} \cdot \frac{\partial \sigma_x}{\partial x}$$

$$\sigma_x = M . \frac{\partial u}{\partial x}$$

or, $$\frac{\partial \sigma_x}{\partial x} = M . \frac{\partial^2 u}{\partial x^2}$$

$$\boxed{\frac{\partial^2 u}{\partial t^2} = \frac{M}{\rho} \cdot \frac{\partial^2 u}{\partial x^2}}$$

$$\frac{M}{\rho} = v_p^2 \quad , \quad v_p \rightarrow \text{Primary (P) wave velocity.}$$

$$\therefore v_p = \sqrt{\frac{M}{\rho}}$$

$$\boxed{\frac{\partial^2 u}{\partial t^2} = v_p^2 \cdot \frac{\partial^2 u}{\partial x^2}}$$